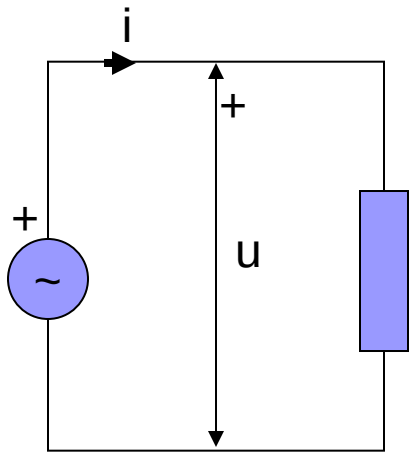
Naizmenične struje

Električna kola naizmenične električne struje

Prosta kola naizmenične struje

- Prosto kolo naizmenične električne struje sastoji se od izvora naizmenične električne struje i jednog potrošača

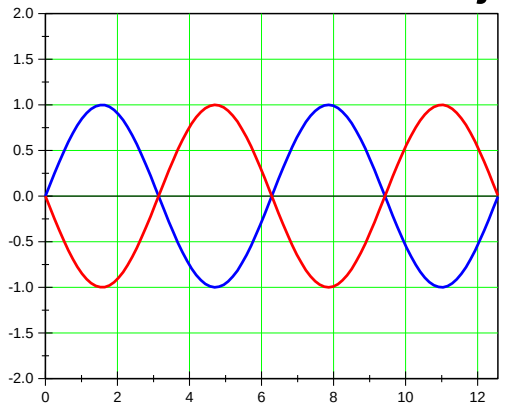


$$u = U\sqrt{2}\cdot\sin(\omega t+\theta) \qquad i = I\sqrt{2}\cdot\sin(\omega t+\psi)$$

- Odrediti vezu između napona na potrošaču i struje koja protiče kroz granu sa potrošačem
- Odrediti veze efektivnih vrednosti (U,I)
- Odrediti veze početnih faza (θ,ψ)

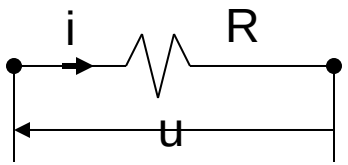
- Smisao referentnog smera struje i napona u kolu naizmenične struje

$$i = I_0\cdot\sin(\omega t)$$
$$-i = -I_0\cdot\sin(\omega t) = I_0\cdot\sin(\omega t+\pi)$$



Prosto kolo sa otpornikom

- Vremensko opisivanje - analitički prikaz



$$i_R = \frac{u_R}{R} \qquad u_R = U\sqrt{2}\cdot\sin(\omega t + \theta)$$

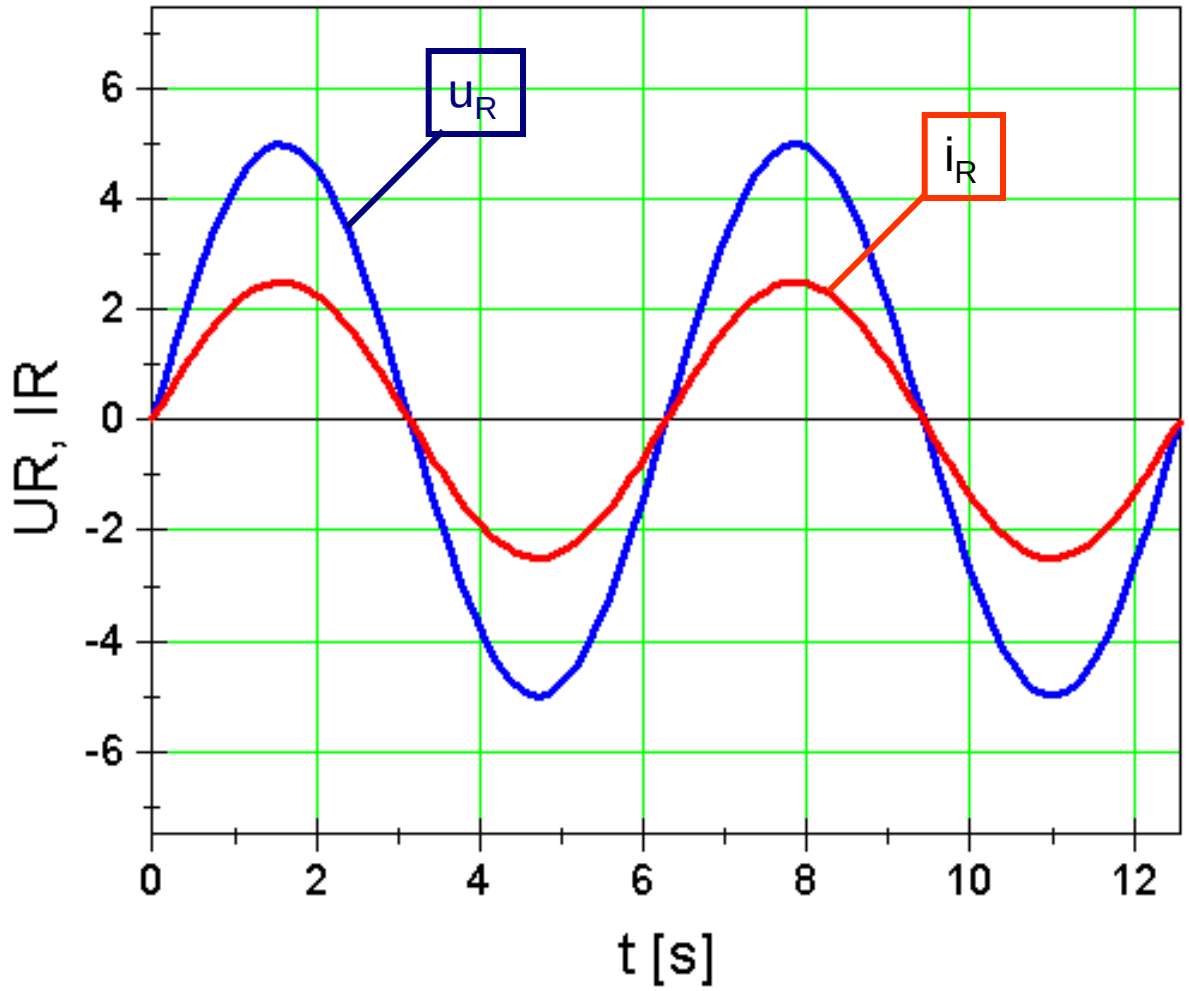
$$i_R = \frac{1}{R}\left[U\sqrt{2}\cdot\sin(\omega t + \theta)\right] = \frac{U}{R}\sqrt{2}\cdot\sin(\omega t + \theta)$$

$$i_R = I\sqrt{2}\cdot\sin(\omega t + \psi)$$

$$I = \frac{U}{R}$$
$$\psi = \theta$$

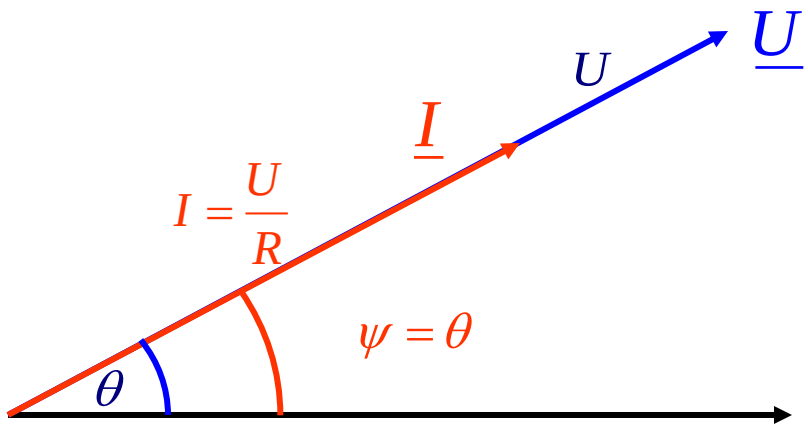
Prosto kolo sa otpornikom

- Vremensko opisivanje - grafički prikaz



Prosto kolo sa otpornikom

- Fazorsko opisivanje



Prosto kolo sa otpornikom

- Kompleksno opisivanje

$$u_C = U\sqrt{2}\cdot\sin(\omega t+\theta) \mapsto \underline{U} = u_1 + ju_2 = U\cos\theta + jU\sin\theta$$
$$i_C = I\sqrt{2}\cdot\sin(\omega t+\psi) \mapsto \underline{I} = i_1 + ji_2 = I\cos\psi + jI\sin\psi$$

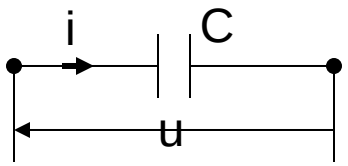
$$\underline{I} = I\cos\psi + jI\sin\psi = \frac{U}{R}\cos\theta + j\frac{U}{R}\sin\theta$$
$$\underline{I} = \frac{1}{R}(U\cos\theta + jU\sin\theta)$$

$$\boxed{\underline{I} = \frac{\underline{U}}{R}}$$

$$\boxed{\underline{U} = R\underline{I}}$$

Prosto kolo sa kondenzatorom

- Vremensko opisivanje - analitički prikaz



$$i_C = C\frac{du_C}{dt} \qquad u_C = U\sqrt{2}\cdot\sin(\omega t+\theta)$$

$$i_C = C\frac{d}{dt}\left[U\sqrt{2}\cdot\sin(\omega t+\theta)\right] = CU\sqrt{2}\cdot\cos(\omega t+\theta)\cdot\omega$$

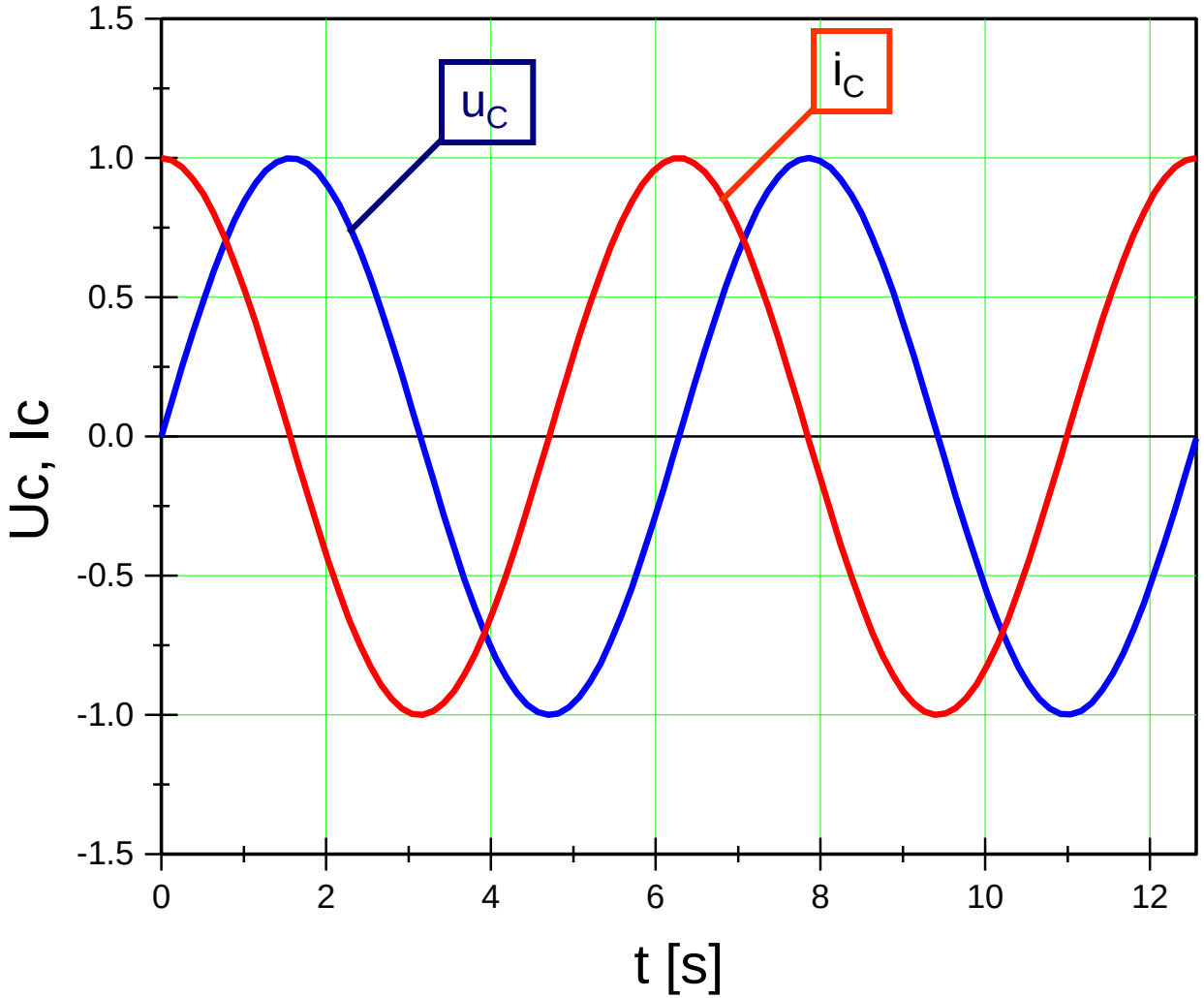
$$i_C = I\sqrt{2}\cdot\sin(\omega t+\psi)$$

$$\cos(\alpha) = \sin(\alpha+\frac{\pi}{2}) \Longrightarrow i_C = \omega CU\sqrt{2}\cdot\sin(\omega t+\theta+\frac{\pi}{2})$$

$$\boxed{\begin{aligned} I &= \omega CU \\ \psi &= \theta+\frac{\pi}{2} \end{aligned}}$$

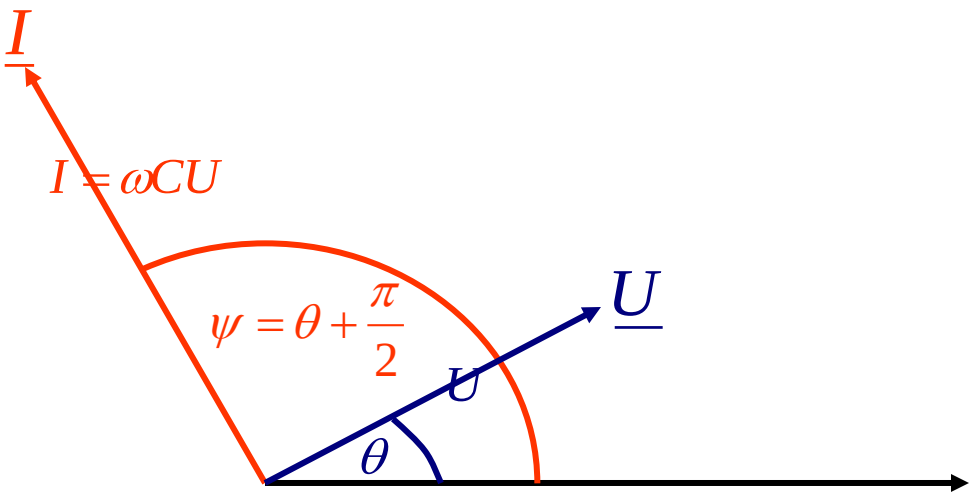
Prosto kolo sa kondenzatorom

- Vremensko opisivanje - grafički prikaz



Prosto kolo sa kondenzatorom

- Fazorski opisivanje



Prosto kolo sa kondenzatorom

- Kompleksno opisivanje

$$u_C = U\sqrt{2}\cdot\sin(\omega t+\theta)\mapsto \underline{U} = u_1 + ju_2 = U\cos\theta + jU\sin\theta$$
$$i_C = I\sqrt{2}\cdot\sin(\omega t+\psi)\mapsto \underline{I} = i_1 + ji_2 = I\cos\psi + jI\sin\psi$$

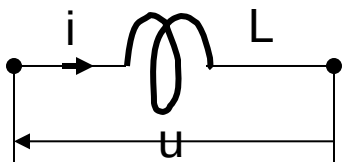
$$\underline{I} = I\cos\psi + jI\sin\psi = \omega CU\cos(\theta+\frac{\pi}{2}) + j\omega CU\sin(\theta+\frac{\pi}{2})$$
$$\underline{I} = -\omega CU\sin\theta + j\omega CU\cos\theta = j\omega C(U\cos\theta + jU\sin\theta)$$

$$\boxed{\underline{I} = j\omega C\underline{U}}$$

$$\boxed{\underline{U} = \frac{1}{j\omega C}\underline{I}}$$

Prosto kolo sa kalemom

- Vremensko opisivanje – analitički prikaz



$$u_L = L\frac{di_L}{dt} \qquad i_L = I\sqrt{2}\cdot\sin(\omega t+\psi)$$

$$u_L = L\frac{d}{dt}\left[I\sqrt{2}\cdot\sin(\omega t+\psi)\right] = LI\sqrt{2}\cdot\cos(\omega t+\psi)\cdot\omega$$

$$u_L = U\sqrt{2}\cdot\sin(\omega t+\theta)$$

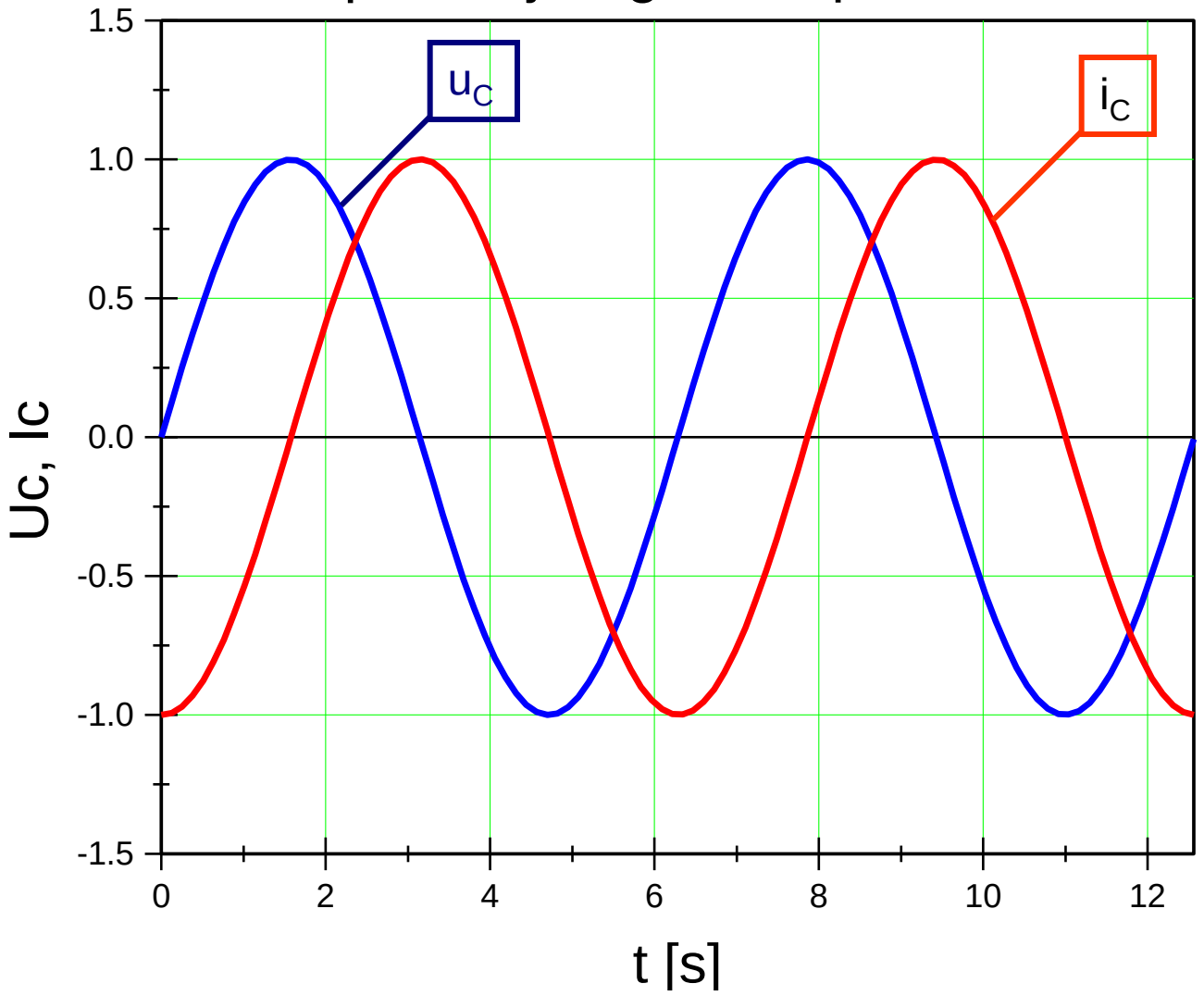
$$\cos(\alpha) = \sin(\alpha+\frac{\pi}{2}) \Longrightarrow u_L = \omega LI\sqrt{2}\cdot\sin(\omega t+\psi+\frac{\pi}{2})$$

$$U = \omega LI$$

$$\theta = \psi + \frac{\pi}{2}$$

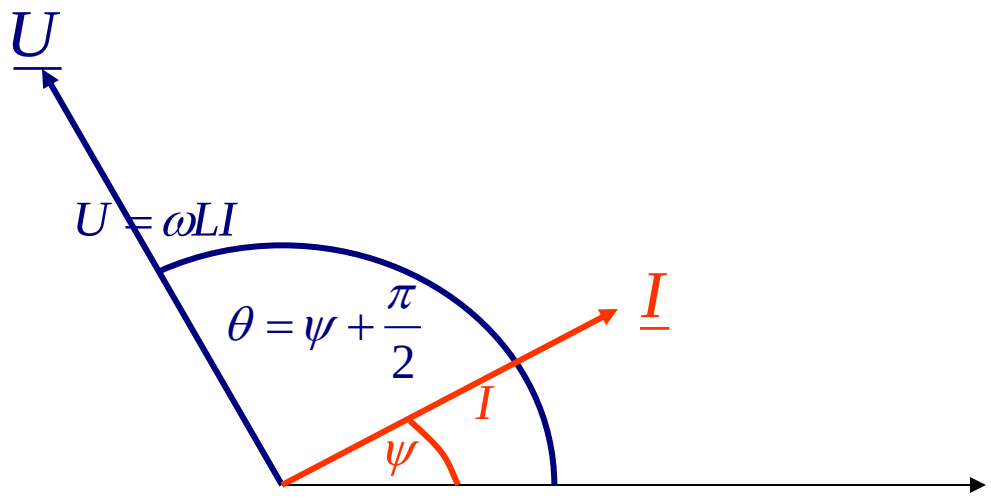
Prosto kolo sa kalemom

- Vremensko opisivanje - grafički prikaz



Prosto kolo sa kalemom

- Fazorsko opisivanje



Prosto kolo sa kalemom

- Kompleksno opisivanje

$$i_C = I\sqrt{2}\cdot\sin(\omega t+\psi) \mapsto \underline{I} = i_1 + ji_2 = I\cos\psi + jI\sin\psi$$
$$u_C = U\sqrt{2}\cdot\sin(\omega t+\theta) \mapsto \underline{U} = u_1 + ju_2 = U\cos\theta + jU\sin\theta$$

$$\underline{U} = U\cos\theta + jU\sin\theta = \omega LI\cos(\psi+\frac{\pi}{2}) + j\omega LI\sin(\psi+\frac{\pi}{2})$$
$$\underline{U} = -\omega LI\sin\psi + j\omega LI\cos\psi = j\omega L(I\cos\psi + jI\sin\psi)$$

$$\boxed{\underline{U} = j\omega L\underline{I}}$$

$$\boxed{\underline{I} = \frac{1}{j\omega L}\underline{U}}$$

Prosta električna kola naizmenične struje

- Veze između napona i struje potrošača

	Vremenski	Efektivne vrednosti	Fazni stav	Kompleksno
R	$u = Ri$	$U = RI$	$\theta = \psi$	$\underline{U} = R\underline{I}$
L	$u = L\frac{di}{dt}$	$U = \omega LI$	$\theta + \frac{\pi}{2} = \psi$	$\underline{U} = j\omega L\underline{I}$
C	$i = C\frac{du}{dt}$	$U = \frac{1}{\omega C}I$	$\theta = \psi + \frac{\pi}{2}$	$\underline{U} = \frac{1}{j\omega C}\underline{I}$

Impedansa

- Efektivne vrednosti napona proporcionalne su efektivnim vrednostima struje
- Slično propocionalnosti napona i struje kod jednosmerne struje
- Odnos napona i struje ima ulogu otpornosti kod jednosmerne struje

$$U = ZI$$

$$\boxed{Z = \frac{U}{I}} \qquad [Z] = \frac{[U]}{[I]} = \frac{V}{A} = \Omega$$

Impedansa električnog elementa je odnos efektivnih vrednosti napona i struje

$$\boxed{Z_R = \frac{U}{I} = R} \qquad \boxed{Z_C = \frac{U}{I} = \frac{1}{\omega C}} \qquad \boxed{Z_L = \frac{U}{I} = \omega L}$$

Impedansa električnog elementa zavisi od frekvence naizmenične struje

Kompleksna impedansa

- Kompleksne vrednosti napona proporcionalne su kompleksnim vrednostima struje

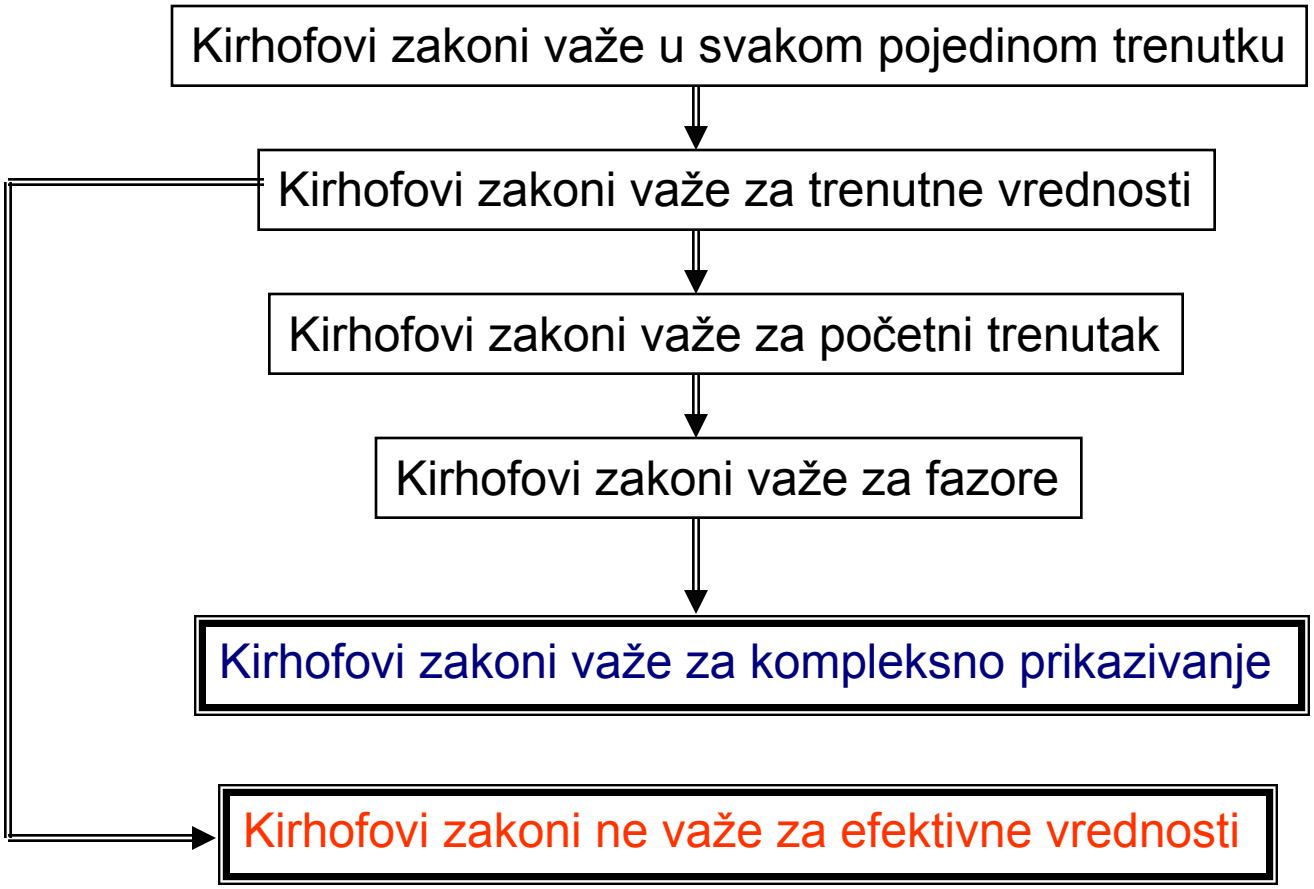
$$\underline{Z} = \frac{\underline{U}}{\underline{I}} \qquad [\underline{Z}] = \frac{[\underline{U}]}{[\underline{I}]} = \frac{V}{A} = \Omega$$

Kompleksna impedansa električnog elementa je odnos kompleksnih vrednosti napona i struje

$$\underline{Z}_R = \frac{\underline{U}}{\underline{I}} = R \qquad \underline{Z}_C = \frac{\underline{U}}{\underline{I}} = \frac{1}{j\omega C} = -\frac{j}{\omega C} \qquad \underline{Z}_L = \frac{\underline{U}}{\underline{I}} = j\omega L$$

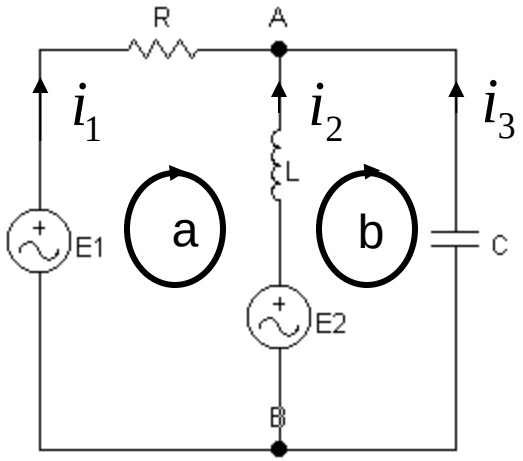
Složena električna kola naizmenične struje

- Osnova za rešavanje električnih kola su Kirhofovi zakoni



Složena električna kola naizmenične struje

- Primer



$$A: \quad \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$a: \quad \underline{E}_1 - \underline{Z}_R \underline{I}_1 + \underline{Z}_L \underline{I}_2 - \underline{E}_2 = 0$$

$$b: \quad \underline{E}_2 - \underline{Z}_L \underline{I}_2 + \underline{Z}_C \underline{I}_3 = 0$$

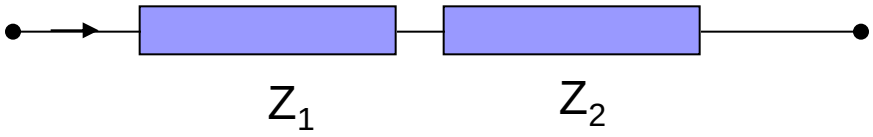
$$A: \quad \underline{I}_1 + \underline{I}_2 + \underline{I}_3 = 0$$

$$a: \quad \underline{E}_1 - R\underline{I}_1 + j\omega L \underline{I}_2 - \underline{E}_2 = 0$$

$$b: \quad \underline{E}_2 - j\omega L \underline{I}_2 - \frac{j}{\omega C} \underline{I}_3 = 0$$

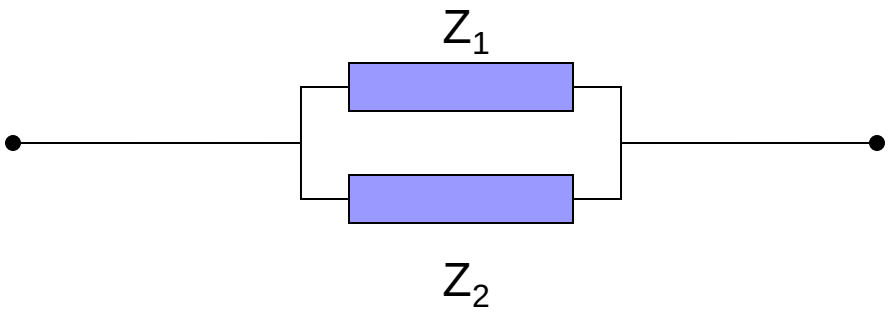
Složena električna kola naizmenične struje

- Vezivanje impedansi
  - redno vezivanje
  - paralelno vezivanje



$$\underline{U} = \underline{U}_1 + \underline{U}_2 \longrightarrow \boxed{\underline{Z} = \underline{Z}_1 + \underline{Z}_2}$$

~~$U = U_1 + U_2 \longrightarrow Z_1 = Z_1 + Z_2$~~

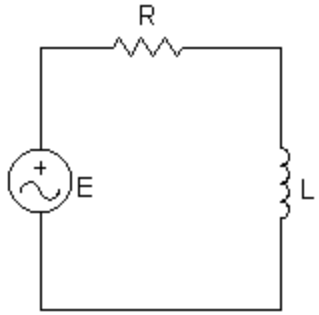


$$\underline{I} = \underline{I}_1 + \underline{I}_2 \longrightarrow \boxed{\frac{1}{\underline{Z}} = \frac{1}{\underline{Z}_1} + \frac{1}{\underline{Z}_2}}$$

~~$I = I_1 + I_2 \longrightarrow \frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$~~

Složena električna kola

- Primer

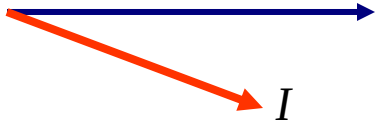


$$E = 220V \quad f = 50\,Hz \quad \omega = 314\,s^{-1} \quad R = 100\,\Omega \quad L = 100\,mH$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} \qquad \underline{Z} = \underline{Z}_R + \underline{Z}_L = R + j\omega L = (100 + j31{,}4)\,\Omega$$

$$\underline{I} = \frac{\underline{U}}{\underline{Z}} = \frac{220\,V}{(100 + j\,31{,}4)\,\Omega} = \frac{220}{100 + j\,31{,}4}\,A \cdot \frac{100 - j\,31{,}4}{100 - j\,31{,}4} = \frac{22000 - j6908}{100^2 + (31{,}4)^2}\,A = \frac{22000 - j6908}{10986}\,A$$

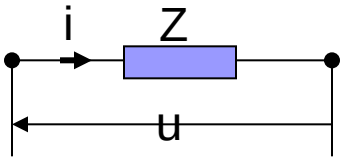
$$\underline{I} = (2 - j0{,}63)A$$



$$I = \sqrt{2^2 + 0{,}63^2}\,A = 2{,}1\,A \qquad Z = \frac{U}{I} = \frac{220V}{2{,}1\,A} = 105\,\Omega$$

$$tg\,\psi = \frac{-0{,}63}{2} \Rightarrow \varphi = -17{,}5^0$$

Kompleksna impedansa potrošača

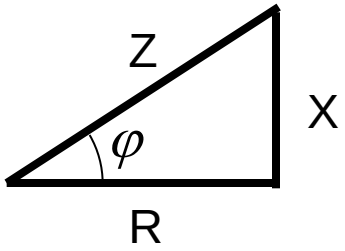


$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = z_1 + jz_2 = R + jX$$ ← reaktansa

$$\underline{Z} = \frac{\underline{U}}{\underline{I}} = \frac{U(\cos\theta + j\sin\theta)}{I(\cos\psi + j\sin\psi)} = \frac{Ue^{j\theta}}{Ie^{j\psi}} = \frac{U}{I}e^{j(\theta-\psi)}$$

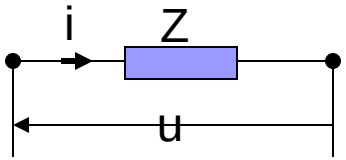
$$\underline{Z} = \frac{U}{I}e^{j(\theta-\psi)} = Ze^{j\varphi} = Z\left(\cos\varphi + j\sin\varphi\right) = R + jX$$

$$\boxed{\varphi = \theta - \psi} \qquad \boxed{R = Z\cdot\cos\varphi} \qquad \boxed{X = Z\cdot\sin\varphi}$$



Trougao impedanse $\varphi_R = 0 \quad \varphi_C = -\frac{\pi}{2} \quad \varphi_L = +\frac{\pi}{2}$

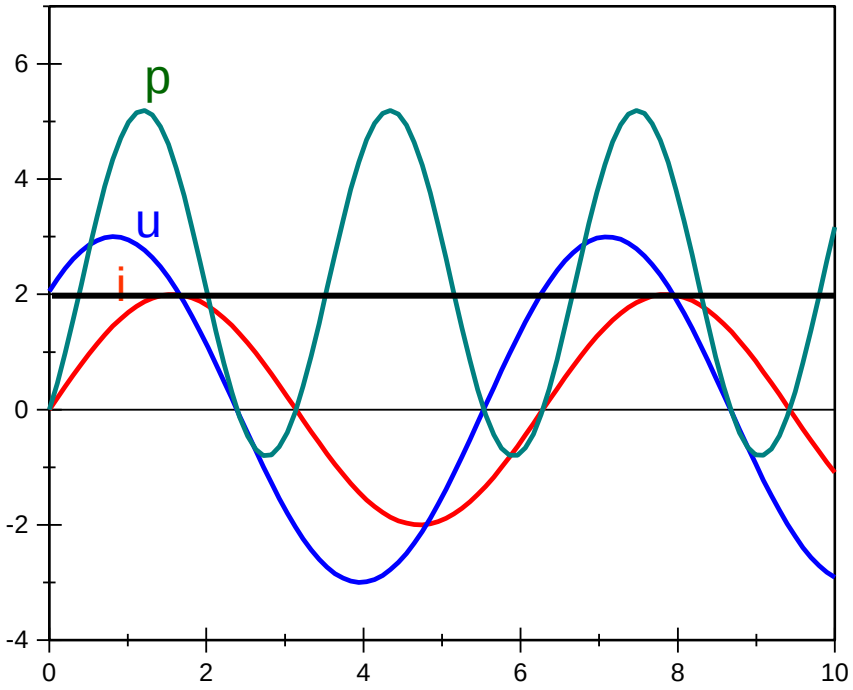
Snaga potrošača naizmenične struje



$$p = ui = 2 \cdot UI \cdot \sin(\omega t)\sin(\omega t + \varphi)$$

$$\sin\alpha \sin\beta = \frac{1}{2}\left[\cos(\alpha - \beta) - \cos(\alpha + \beta)\right]$$

$$p = ui = UI\cos\varphi - UI\cos(2\omega t + \varphi)$$

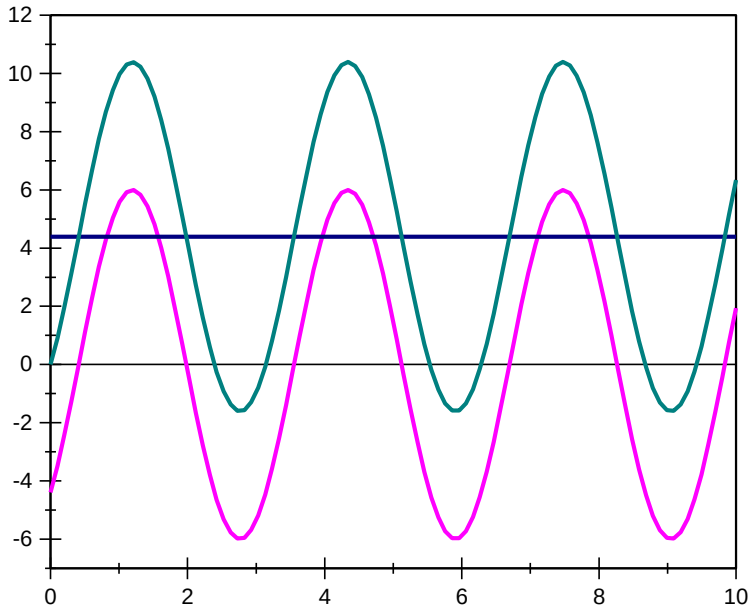


$$\boxed{P = UI\cos\varphi} \qquad [P] = W$$

Srednja ili **aktivna** snaga

Snaga naizmenične struje

- Reaktivna snaga

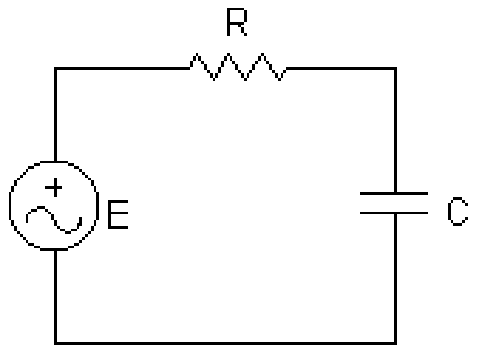


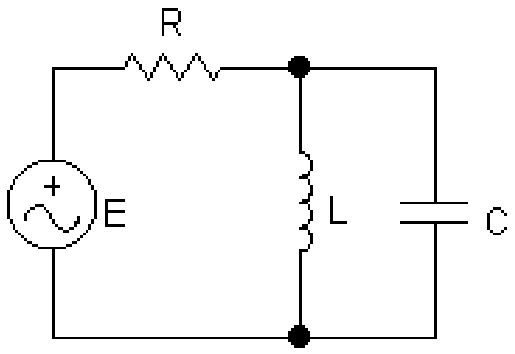
- Snaga osciluje sa srednjom vrednošću UIcos(φ) i amplitudom UI
- Električni element nekad radi kao potrošač, a nekada kao generator
- Električni element u delu perioda prima energiju od izvora, a u delu perioda vraća energiju izvoru
- Deo energije koji se vraća izvoru je reaktivna energija
- Srednja vrednost reaktivne energije je nula

Amplituda reaktivne snage $\boxed{Q = UI\sin\varphi}$ $[Q] = VAr$

Snaga naizmenične struje

- Reaktivna snaga
  - nepovoljna pojava u kolima naizmenične struje
  - prenos reaktivne snage u oba smera je rasipanje energije
  - mera energetskog kvaliteta elementa je **faktor snage** $\cos\varphi$
  - naplaćuje se samo aktivna snaga
  - distributeri električne energije uslovljavaju snabdevanje visokom vrednošću faktora snage
  - postupak povećanja faktora snage se naziva **kompenzacija**



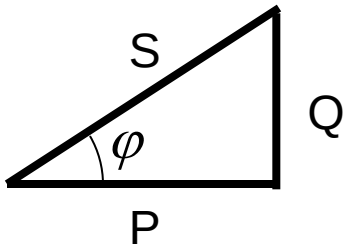


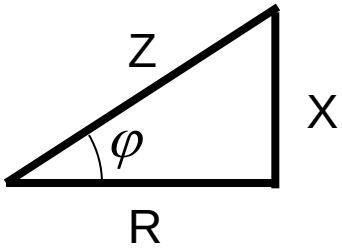
Snaga naizmenične struje

- Trougao snage

$$P = UI\cos\varphi \qquad Q = UI\sin\varphi \qquad S = \sqrt{P^2 + Q^2} = UI$$

prividna snaga





Trougao snage i trougao impedanse su slični

$$S = UI = ZI^2$$

$$P = UI\cos\varphi = ZI^2\cos\varphi = RI^2$$

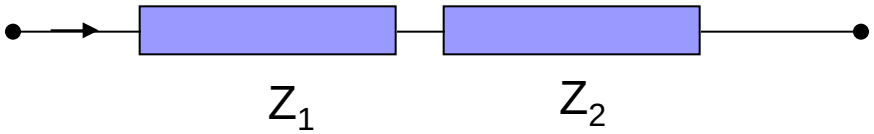
$$Q = UI\sin\varphi = ZI^2\sin\varphi = XI^2$$

Snaga naizmenične struje

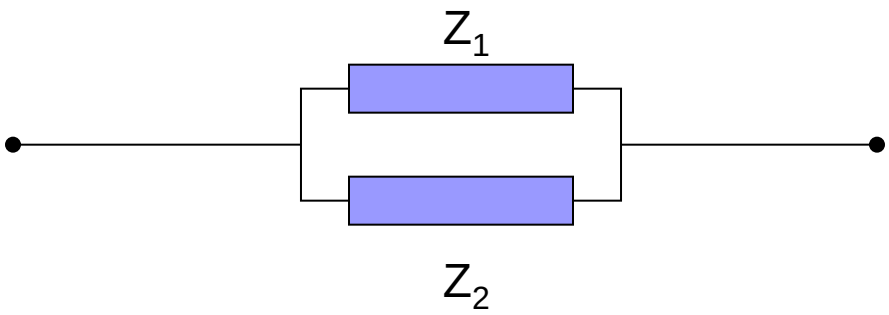
- Kompleksna snaga

$$\underline{S} = \underline{U}\,\underline{I}^{*}$$

$$\underline{S} = \underline{U}\,\underline{I}^{*} = \underline{Z}\,\underline{I}\,\underline{I}^{*} = \underline{Z}I^{2} = RI^{2} + jXI^{2} = P + jQ$$



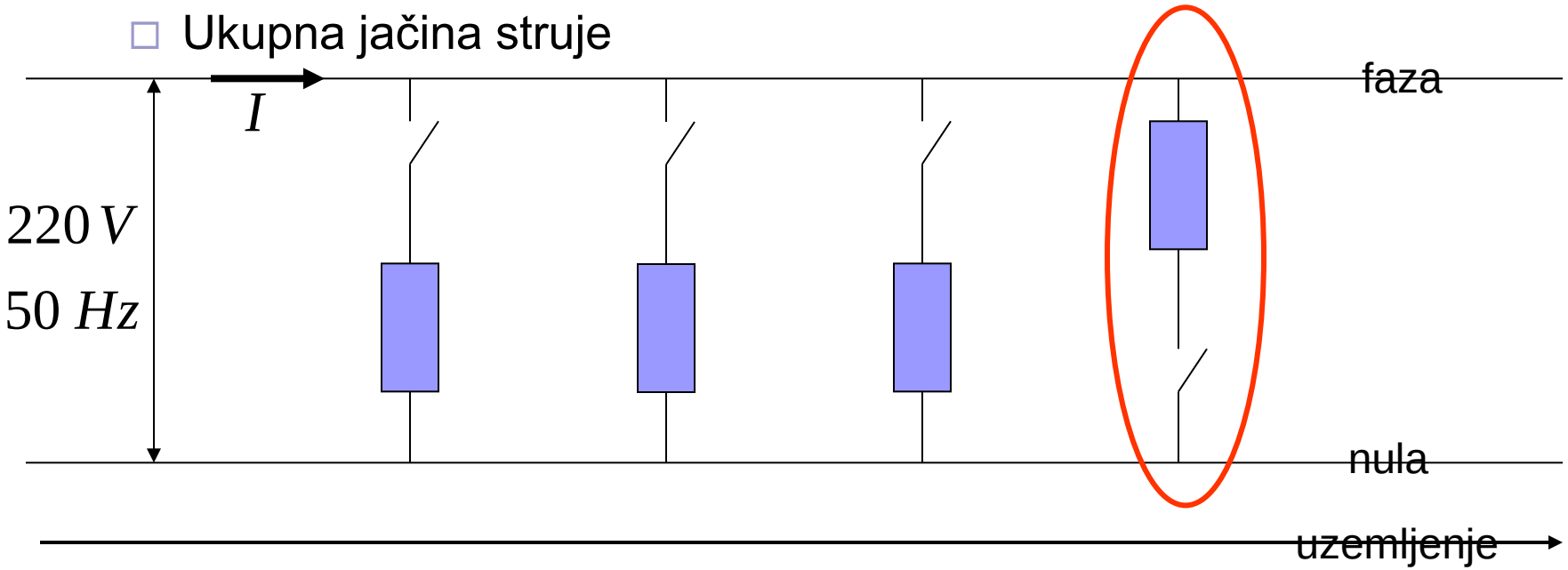
$$\underline{S} = \underline{U}\,\underline{I}^{*} = (\underline{U}_1 + \underline{U}_2)\underline{I}^{*} = \underline{U}_1\underline{I}^{*} + \underline{U}_2\underline{I}^{*} = \underline{S}_1 + \underline{S}_2$$



$$\underline{S} = \underline{U}\,\underline{I}^{*} = \underline{U}(\underline{I}_1^{*} + \underline{I}_2^{*}) = \underline{U}\,\underline{I}_1^{*} + \underline{U}\,\underline{I}_2^{*} = \underline{S}_1 + \underline{S}_2$$

Snaga naizmenične struje

- Proračun električne instalacije
  - Ukupna potrošnja električne energije
  - Ukupna jačina struje



$$\underline{S} = (\underline{S}_1 + \underline{S}_2 + \ldots + \underline{S}_n) = (P_1 + P_2 + \ldots + P_n) + j \cdot (Q_1 + Q_2 + \ldots + Q_n)$$

$$S = \sqrt{(P_1 + P_2 + \ldots + P_n)^2 + (Q_1 + Q_2 + \ldots + Q_n)^2}$$

$$S = UI \Rightarrow I = \frac{S}{U}$$

Snaga naizmenične struje

- Aktivna snaga se pretvara u toplotu i mehanički rad i plaćamo je
- Reaktivnu snagu vraćamo izvorima električne energije i ne plaćamo je
- Prividna snaga se koristi za dimenzionisanje električnih instalacija